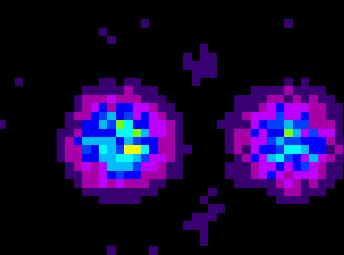


# Quantum logic gates between trapped ions

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Christopher Monroe  
FOCUS Center & Department of Physics  
University of Michigan

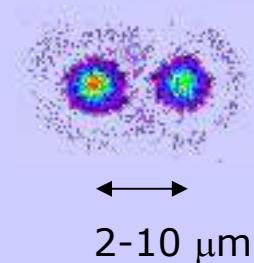


# Quantum Information and Trapped Ions

$$H = \sum_{i=1}^N \frac{1}{2} \omega_i(t) \hat{\vec{\sigma}}^{(i)} + \sum_{i,j=1}^N g_{ij}(t) \hat{\vec{\sigma}}^{(i)} \cdot \hat{\vec{\sigma}}^{(j)}$$

↑  
N qubits

↑  
controlled  
coupling



phonons: Coulomb interaction  
photons: Linear optics, cavity-QED

# PERIODIC TABLE

## Atomic Properties of the Elements

Group IA																		
1	H																	
2	Li																	
3	Na																	
4	Mg																	
5	K																	
6	Rb																	
7	Fr																	
8	Ba																	
9	Ra																	

Frequently used fundamental physical constants																	
1 second = 9 192 631 770 periods of radiation corresponding to the transition between two hyperfine levels of the ground state of $^{133}\text{Cs}$																	
speed of light in vacuum																	
$c = 299\,792\,458 \text{ m s}^{-1}$ (exact)																	
Planck constant																	
$\hbar = 6.6261 \times 10^{-34} \text{ J s}$ ( $\hbar = h/\pi$ )																	
elementary charge																	
$e = 1.6022 \times 10^{-19} \text{ C}$																	
electron mass																	
$m_e = 9.1094 \times 10^{-31} \text{ kg}$																	
proton mass																	
$m_p = 1.6726 \times 10^{-27} \text{ kg}$																	
fine-structure constant																	
$\alpha = 1/137.036$																	
Rydberg constant																	
$R_{\infty} = 10.973\,732 \text{ m}^{-1}$																	
Boltzmann constant																	
$k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1}$																	

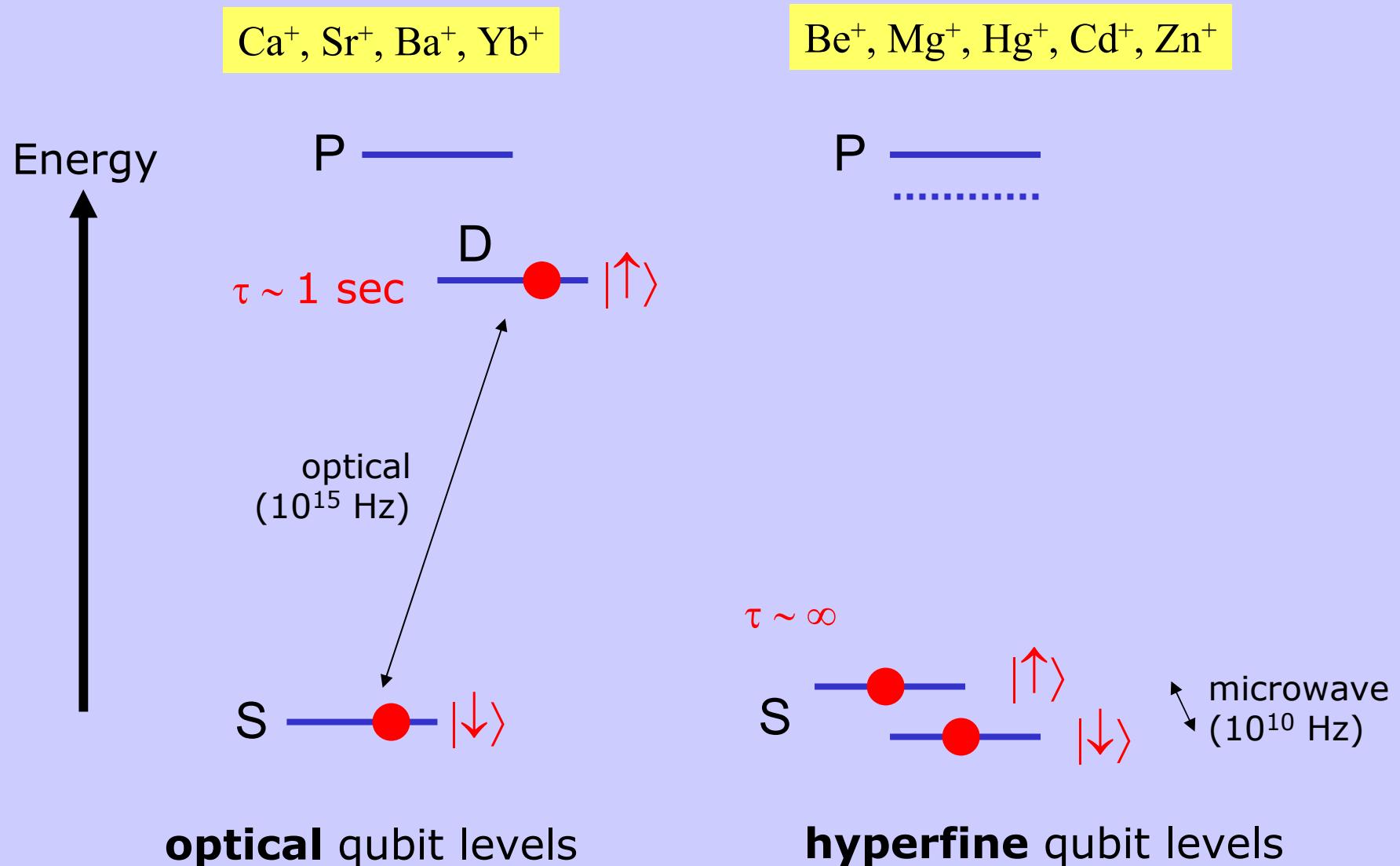
Physics Laboratory **NIST** Standard Reference Data Program  
physics.nist.gov www.nist.gov

U.S. DEPARTMENT OF COMMERCE  
Technology Administration  
National Institute of Standards and Technology

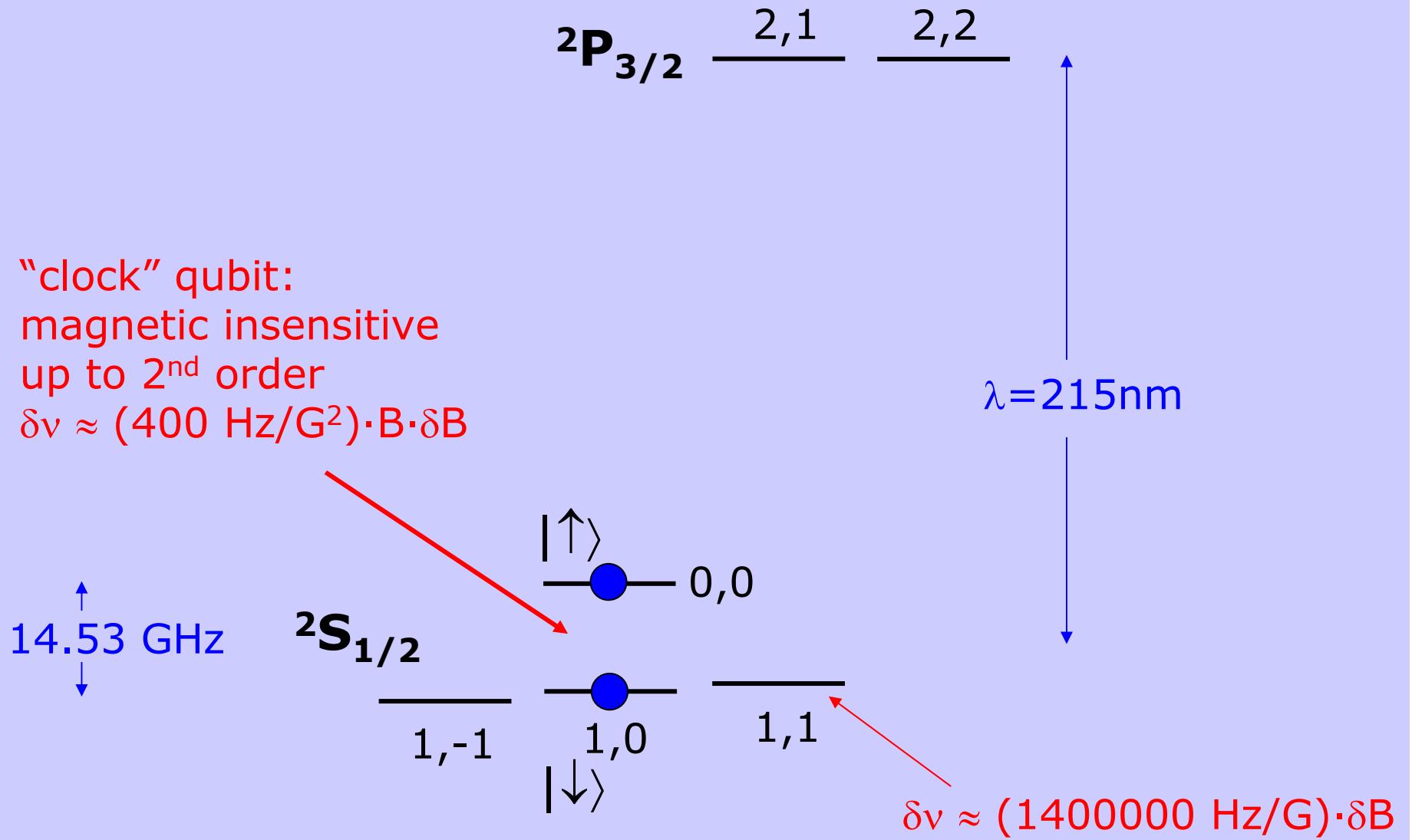
VIII	He	Ne	Ar	Kr	Rn
	Helium	Neon	Argon	Krypton	Radon
	4.00260	21.1797	39.904	83.80	(227)
	$10^{-3}$	$10^{-3}$	$10^{-3}$	$10^{-3}$	$10^{-3}$
	24.5874	21.0626	12.1285	10.7465	9.1020
IIIA	IVA	VA	VIA	VIIA	VIIIA
Sc	Ti	V	Cr	Mn	Fe
Ca	Titanium	Vanadium	Chromium	Manganese	Iron
Scandium	44.95591	50.9415	51.9861	54.93895	55.845
Calcium	40.078	(40.078)	(40.078)	(40.078)	(40.078)
[Ar]4s <sup>2</sup>					
6.1132	6.8281	6.7462	7.4340	7.3824	7.8810
Sc	Zr	Nb	Tc	Ru	Pd
Y	Zirconium	Niobium	Techneium	Ruthenium	Palladium
Yttrium	91.224	92.90838	(98)	101.07	108.42
[Kr]4s <sup>2</sup>					
5.639	6.6339	6.7389	7.28	7.3805	7.5989
Ba	Rb	Mo	Tc	Ru	Pd
Radium	137.1327	Molybdenum	Techneium	Ruthenium	Palladium
[Kr]4s <sup>2</sup>					
5.2117	8.2171	6.6339	7.821	7.8810	8.3389
Hf	Ta	W	Re	Os	Ir
Tantalum	160.9479	Tungsten	Rhenium	Dysprosium	Pt
178.49	[Kr]4f <sup>15</sup> 5d <sup>2</sup>	163.84	166.207	150.23	155.076
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
6.8201	7.6496	7.8640	8.4382	8.4382	8.8687
Ta	Ru	Os	Ir	Pt	Au
Rhenium	166.207	Rhenium	Iridium	Platinum	Gold
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
7.6335	7.8640	8.4382	8.8687	9.2265	9.4375
Os	Ir	Pt	Au	Hg	Tl
Dysprosium	Iridium	Platinum	Gold	Mercury	Thallium
150.23	162.217	166.207	169.96868	203.89	204.3833
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
8.4382	8.8687	9.2265	9.4375	10.403	10.403
Ir	Pt	Au	Hg	Tl	Pb
Tellurium	Palladium	Mercury	Thallium	Lanthanum	Lead
162.217	166.207	169.96868	203.89	139.9055	207.2
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
8.8687	9.2265	9.4375	10.403	5.5387	10.403
Pt	Au	Hg	Tl	Ce	Ac
Palladium	Mercury	Thallium	Lanthanum	Cerium	Actinium
166.207	169.96868	203.89	139.9055	140.116	(227)
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
9.2265	9.4375	10.403	10.403	5.5387	5.5387
U	Np	Pu	Am	Cm	Bk
Uranium	Neptunium	Plutonium	Americium	Curium	Berkelium
236.0289	[Xe]4f <sup>15</sup> 5d <sup>2</sup>				
6.1941	6.2557	6.0262	5.9735	6.1077	6.1979
Np	Pu	Am	Cm	Bk	Cf
Neptunium	Plutonium	Americium	Curium	Berkelium	Californium
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
6.2557	6.0262	5.9735	6.1077	6.1979	6.42
Am	Cm	Bk	Cf	Fm	Md
Americium	Curium	Berkelium	Californium	Berkelium	Mendelevium
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
5.9735	6.1077	6.1979	6.42	6.56	6.65
Cf	Fm	Md	No	Lr	Lu
Californium	Berkelium	Mendelevium	Nobelium	Lawrencium	Luxium
[Xe]4f <sup>15</sup> 5d <sup>2</sup>					
6.42	6.56	6.65	6.87	4.87	5.4276

Yb	Tb	Dy	Ho	Er	Tm	Yb	Lu
Ytterbium	Terbium	Dysprosium	Holmium	Erbium	Thulium	Ytterbium	Luxium
173.04	167.26	162.50	164.93032	167.26	169.96868	173.04	174.907
[Xe]4f <sup>14</sup> 5s <sup>2</sup>							
6.1941	6.1979	6.1077	6.1077	6.1979	6.42	6.56	6.65
Np	Pu	Am	Cm	Bk	Cf	Es	Md
Neptunium	Plutonium	Americium	Curium	Berkelium	Californium	Berkeleyium	Mendelevium
[Xe]4f <sup>15</sup> 5d <sup>2</sup>							
5.9735	6.0262	5.9735	6.1077	6.1979	6.42	6.56	6.65
Cm	Bk	Cf	Es	Fm	M		

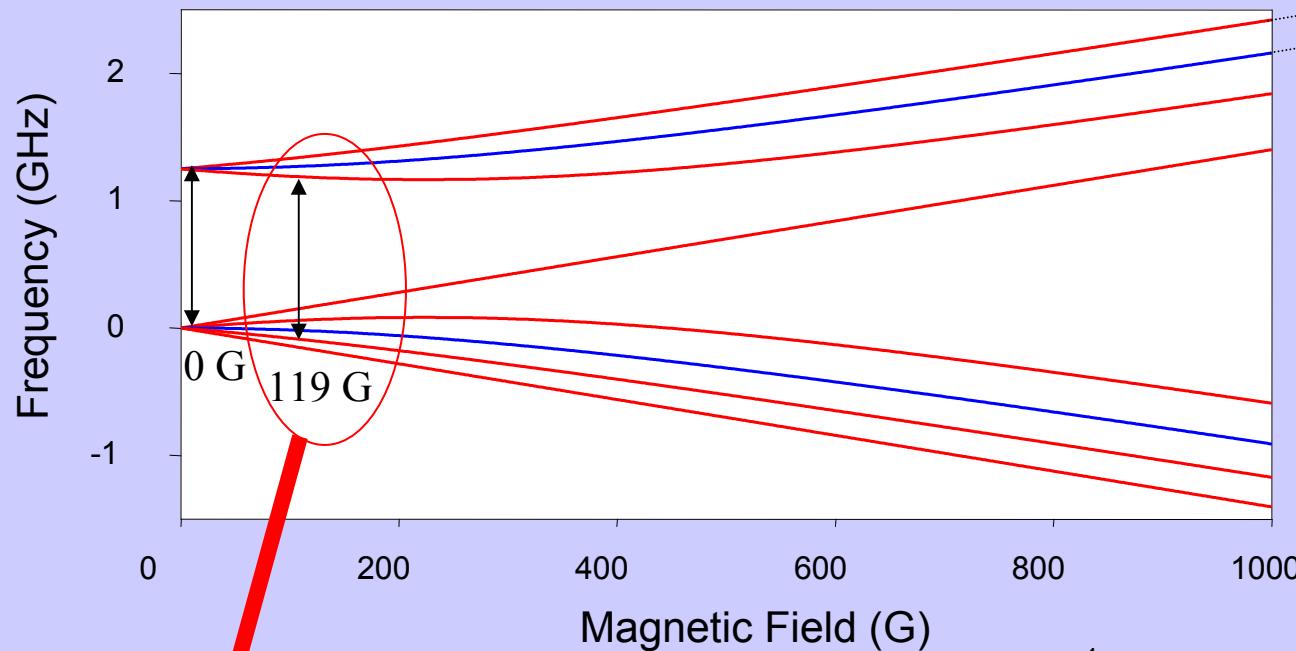
# Atomic Ion Internal Energy Levels



# $^{111}\text{Cd}^+$ atomic structure

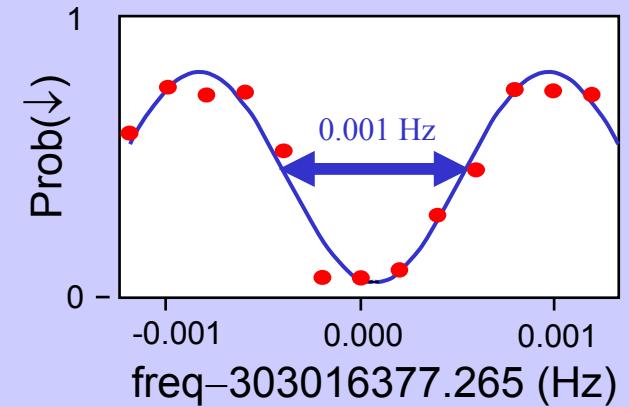


# “Clock” qubits: ground states of ${}^9\text{Be}^+$

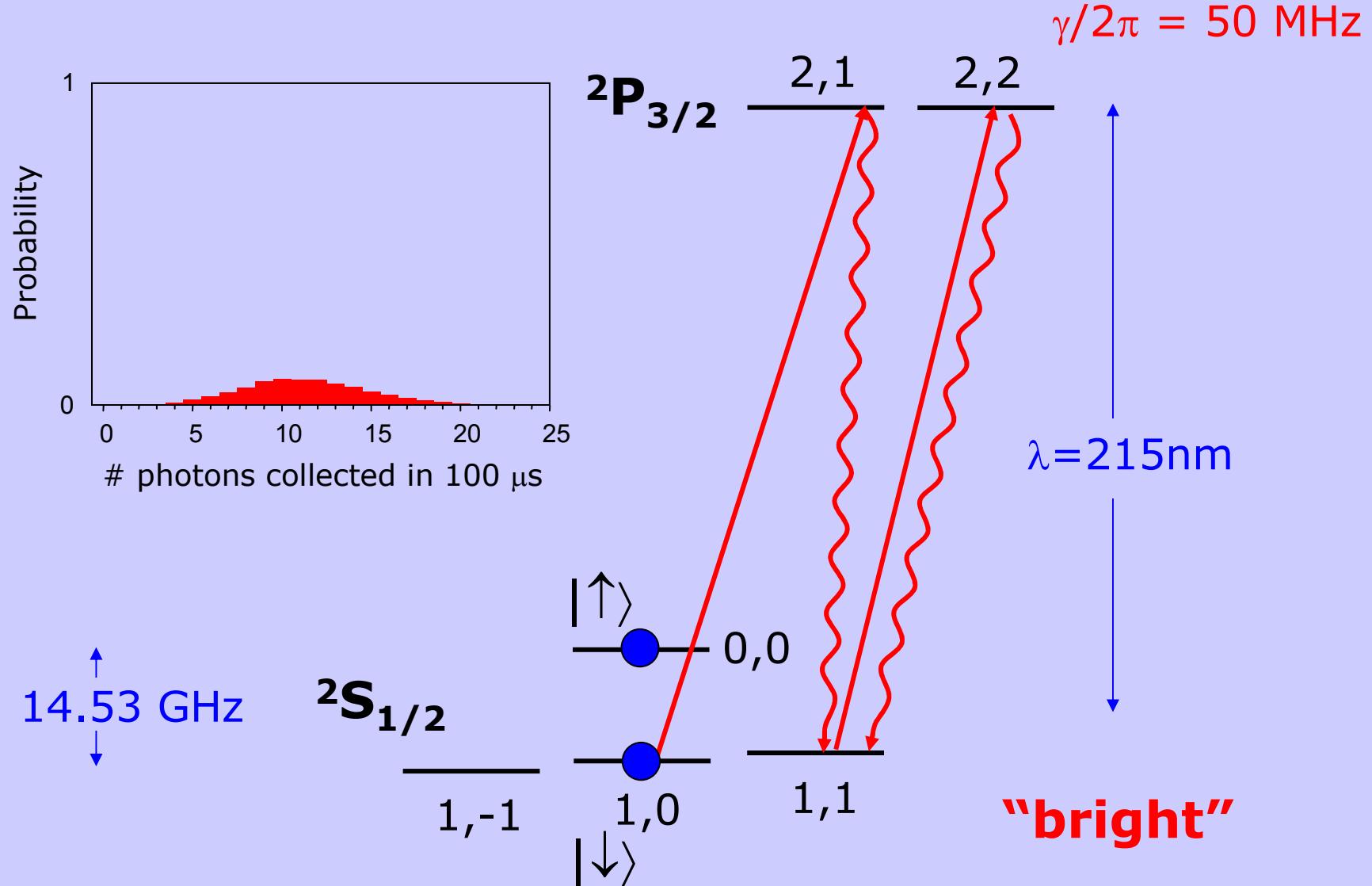


Langer, et al., PRL **95**, 060502 (2005)  
**POSTERS M01,M08,M14,M21**

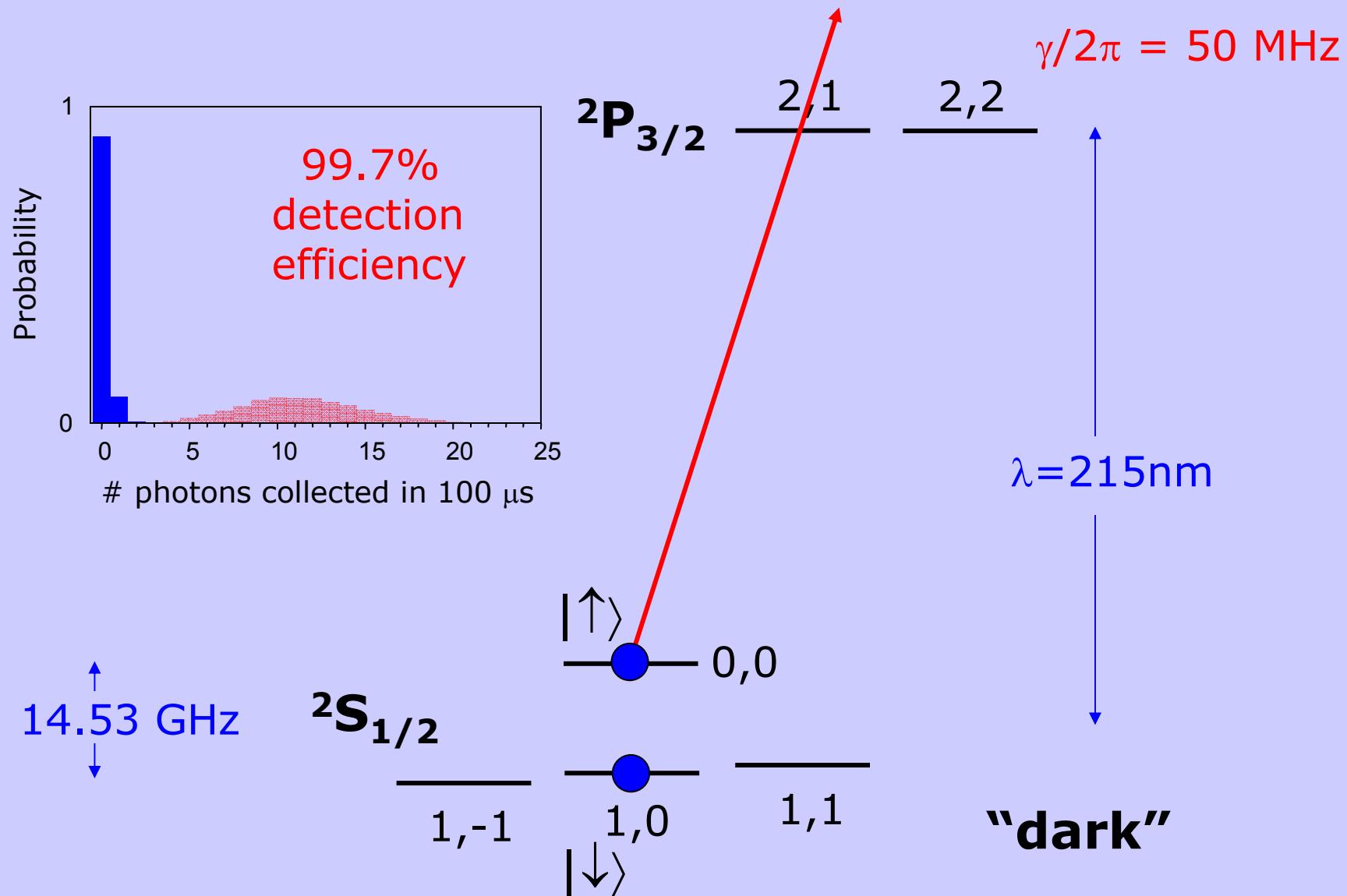
Bollinger, et al.,  
IEEE Trans. Inst. Meas. **40**, 126 (1991)



# qubit measurement



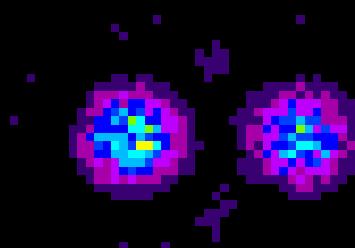
# qubit measurement



# Entangling Gate Schemes for Trapped Ion

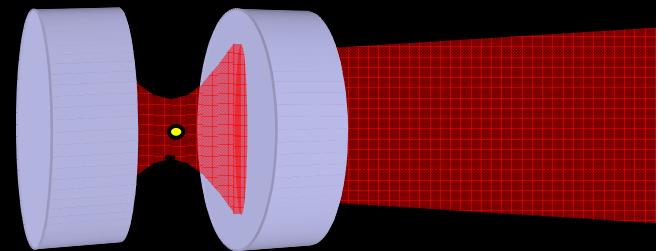
## MOTIONAL GATES

- Direct Phonon Coupling
- Spin-dependent Forces



## PHOTONIC GATES

- Linear Optics (probabilistic)
- Cavity-QED



# Optical Raman transitions

$\Delta < \Delta_{FS}$

$$\Omega = \frac{g_1 g_2}{\Delta}$$

$$N = \frac{\Omega}{\gamma_{sp}} \sim \frac{\Delta}{\gamma}$$

$$\gamma_{sp} = \frac{\gamma(g_1^2 + g_2^2)}{\Delta^2}$$

Heinzen, et al., PRA **42**, 2977 (1990)

C.M., et al., PRL **75**, 4011 (1995)

$\Delta >> \Delta_{FS}$

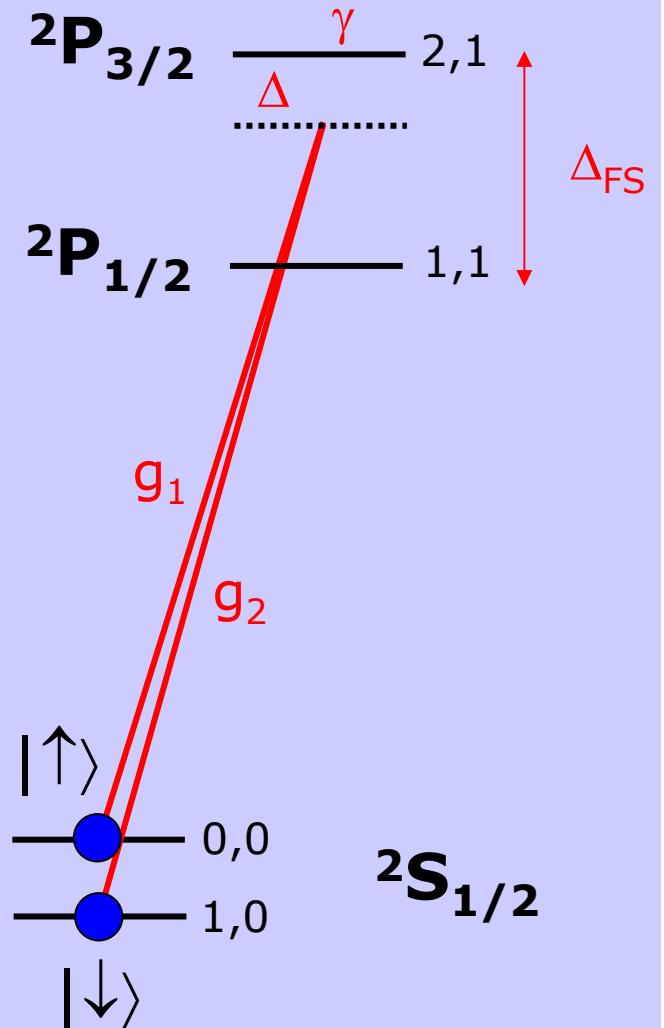
$$\Omega = \frac{g_1 g_2 \Delta_{FS}}{\Delta^2}$$

$$N = \frac{\Omega}{\gamma_{sp}} \sim \frac{\Delta^2}{\gamma \Delta_{FS}}$$

$$\gamma_{sp} = \frac{\gamma(g_1^2 + g_2^2) \Delta_{FS}^2}{\Delta^4}$$

Ozeri, et al., PRL **95**, 030403 (2005)

**POSTER M20**



# Spin-motion coupling: some math

$$H = \hbar\omega_0\hat{\sigma}_z + \underbrace{\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2}_{\hbar\omega(a^+a + 1/2)} - \hat{\mu} \cdot E(\hat{x})$$

frequency of applied radiation

$$- \mu_0 \cdot \frac{E_0}{2} (\hat{\sigma}_+ + \hat{\sigma}_-) (e^{ik\hat{x}-i\omega_L t} + e^{-ik\hat{x}+i\omega_L t})$$

interaction frame; “rotating wave approximation”

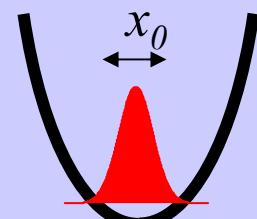
$$H = \hbar g (\hat{\sigma}_+ e^{ik\hat{x}-i\delta t} + \hat{\sigma}_- e^{-ik\hat{x}+i\delta t})$$

$\delta = (\omega_2 - \omega_1) - \omega_{HF}$  = detuning of beat note

$k = k_2 - k_1$  = wavevector difference

$$\hat{x} = x_0 (a e^{-i\omega t} + a^+ e^{i\omega t})$$

$$x_0 = \sqrt{\frac{\hbar}{2m\omega}}$$



$$H = \hbar g [\hat{\sigma}_+ e^{i\eta(ae^{-i\omega t} + a^+ e^{i\omega t}) - i\delta t} + \hat{\sigma}_- e^{-i\eta(ae^{-i\omega t} + a^+ e^{i\omega t}) + i\delta t}]$$

$\eta \equiv kx_0$  = “Lamb-Dicke” parameter  
 $\eta\sqrt{n+1} \ll 1$ : “Lamb-Dicke limit”

stationary terms arise in  $H$  at particular values of  $\delta$  (for  $g \ll \omega$ ):

$$\delta = 0 \quad H_0 = \hbar g (\hat{\sigma}_+ + \hat{\sigma}_-) \longrightarrow \langle \downarrow, n | H_0 | \uparrow, n \rangle = \hbar g$$

“CARRIER”

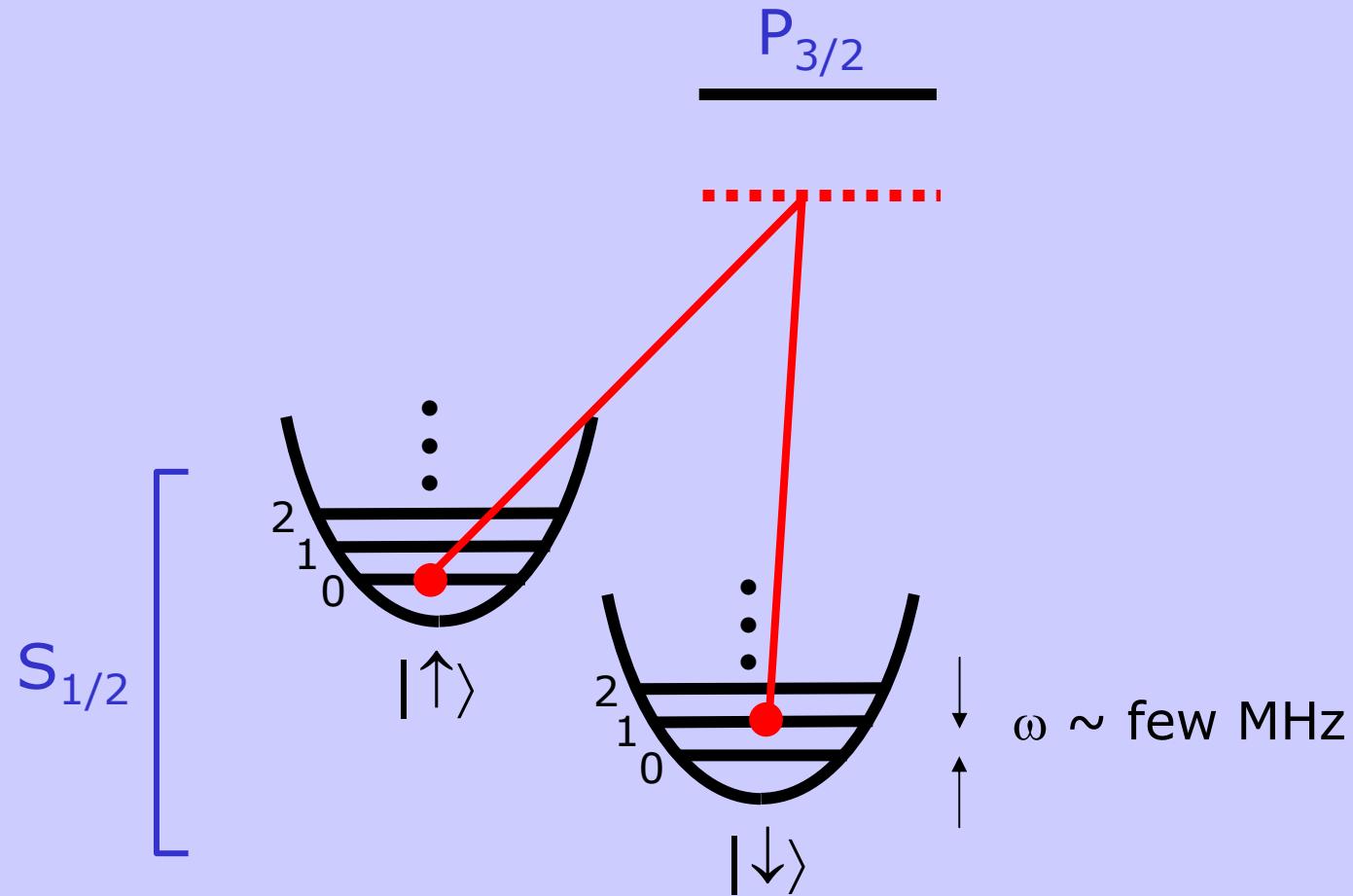
$$\delta = -\omega \quad H_{-1} = \hbar g \eta (\hat{\sigma}_+ a^+ + \hat{\sigma}_- a) \longrightarrow \langle \downarrow, n+1 | H_{-1} | \uparrow, n \rangle = \hbar g \eta \sqrt{n+1}$$

“1<sup>ST</sup> RED SIDEBAND”

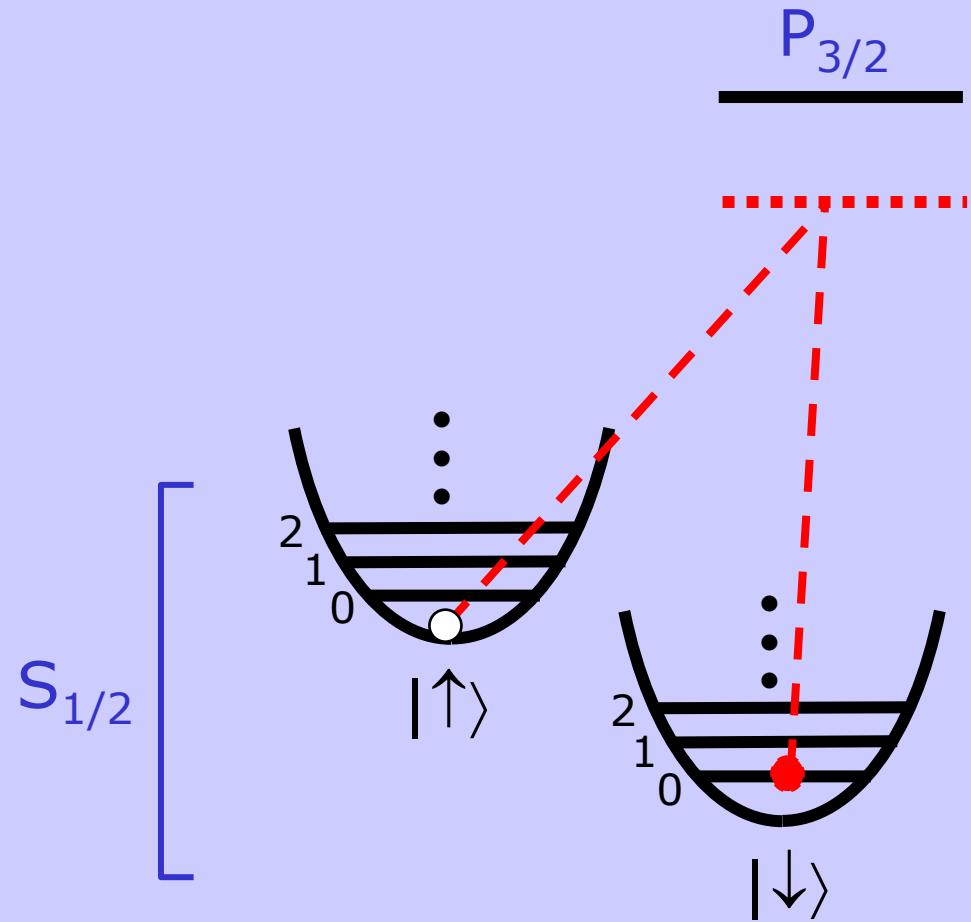
$$\delta = +\omega \quad H_{+1} = \hbar g \eta (\hat{\sigma}_+ a + \hat{\sigma}_- a^+) \longrightarrow \langle \downarrow, n-1 | H_0 | \uparrow, n \rangle = \hbar g \eta \sqrt{n}$$

“1<sup>ST</sup> BLUE SIDEBAND”

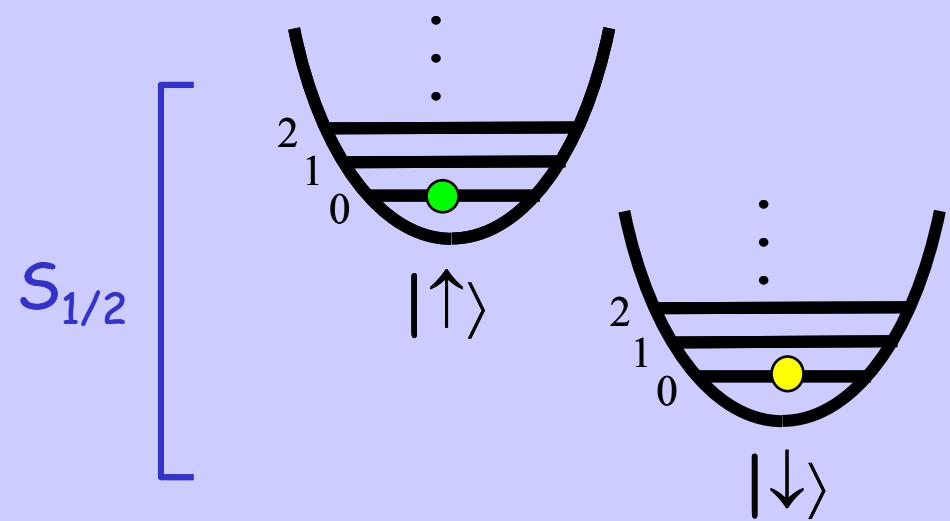
excitation on 1<sup>st</sup> lower ("red") motional sideband (n=0)



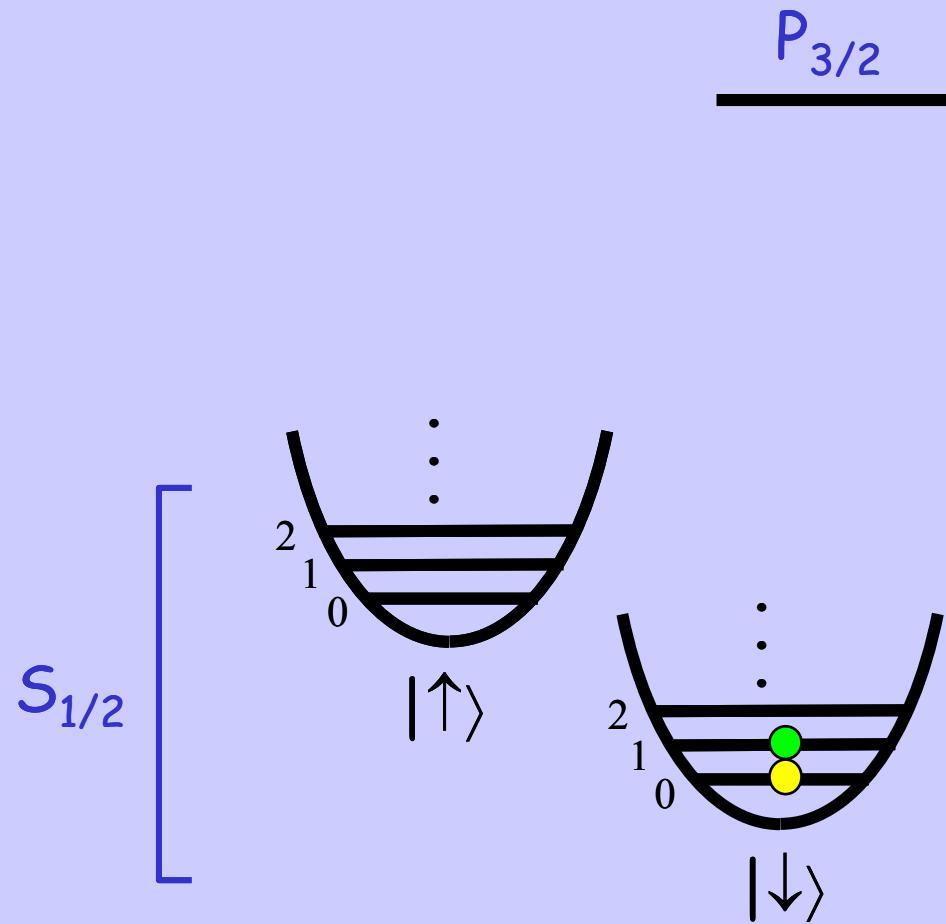
excitation on 1<sup>st</sup> lower ("red") motional sideband (n=0)



$P_{3/2}$

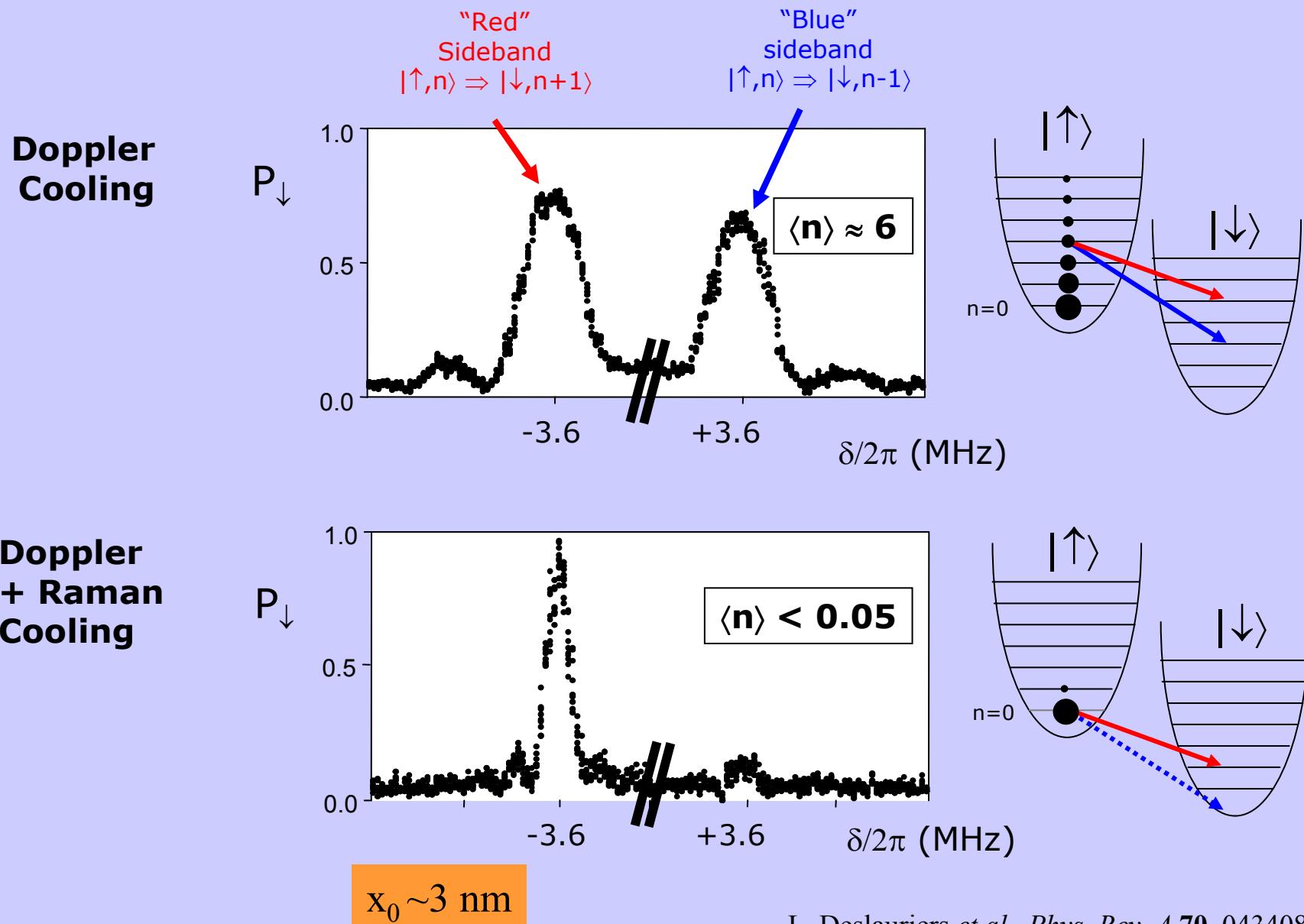


Mapping:  $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)$   $|0\rangle_m \rightarrow |\downarrow\rangle (\alpha|0\rangle_m + \beta|1\rangle_m)$



Mapping:  $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)|0\rangle_m \rightarrow |\downarrow\rangle (\alpha|0\rangle_m + \beta|1\rangle_m)$

# Raman spectrum of single $^{111}\text{Cd}^+$ ion (3.6 MHz trap)



# Universal Quantum Logic Gates with Trapped Ions

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

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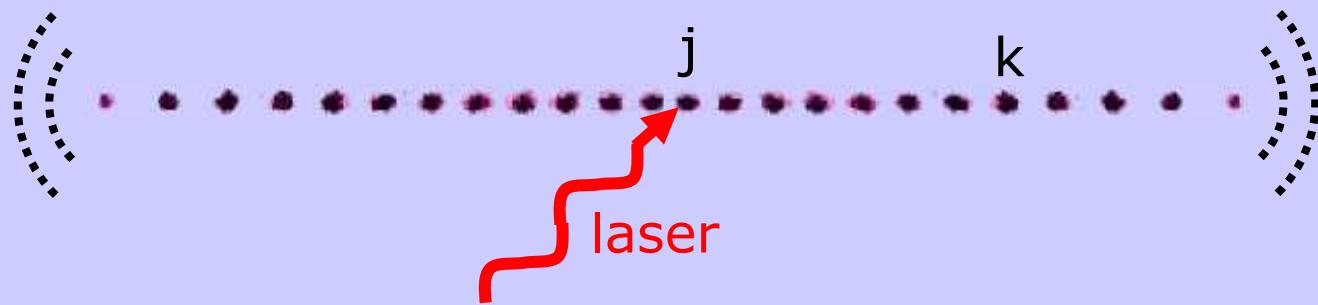
$n=0$

**Step 1** Laser cool collective motion to rest

# Universal Quantum Logic Gates with Trapped Ions

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

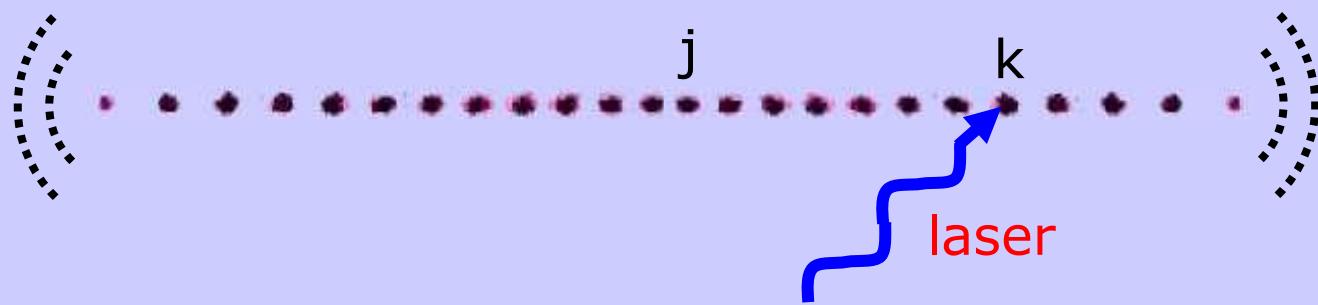
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**Step 2** Map  $j^{\text{th}}$  qubit to collective motion

# Universal Quantum Logic Gates with Trapped Ions

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)



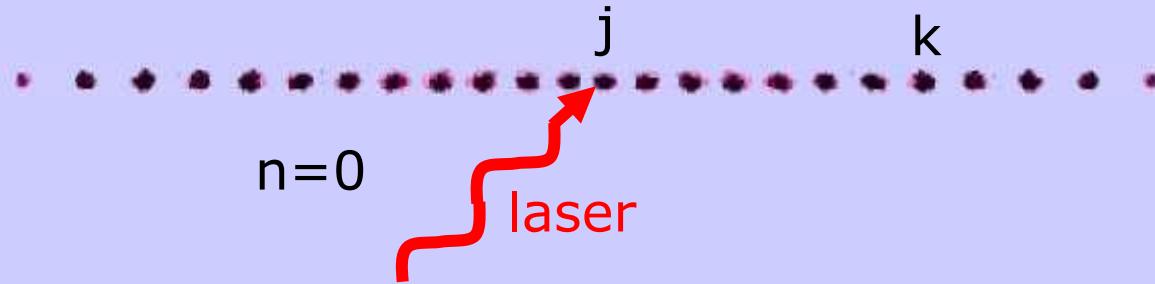
**Step 3** Flip  $k^{\text{th}}$  qubit depending upon motion



# Universal Quantum Logic Gates with Trapped Ions

Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

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**Step 4** Remap collective motion to  $j^{\text{th}}$  qubit  
(reverse of Step 1)

Net result:  $[|\downarrow\rangle_j + |\uparrow\rangle_j] |\downarrow\rangle_k \rightarrow |\downarrow\rangle_j |\downarrow\rangle_k + |\uparrow\rangle_j |\uparrow\rangle_k$

# Demonstrations of Cirac-Zoller '95 Gates

- CNOT between motion and spin (1 ion)

C.M., et al., *Phys. Rev. Lett.* **75**, 4714 (1995)

- CNOT between spins of 2 ions

Schmidt-Kaler, et al., *Nature* **422**, 408-411 (2003).

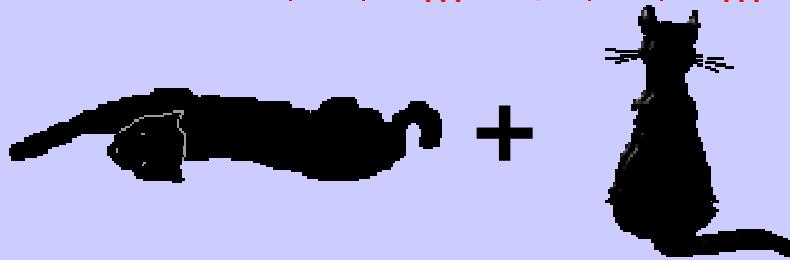
- Entangled “W-states” between 3-8 ions

Roos, et al., *Science* **304**, 1478 (2004)

Häffner, et al., *Nature* **438**, 643 (2005)

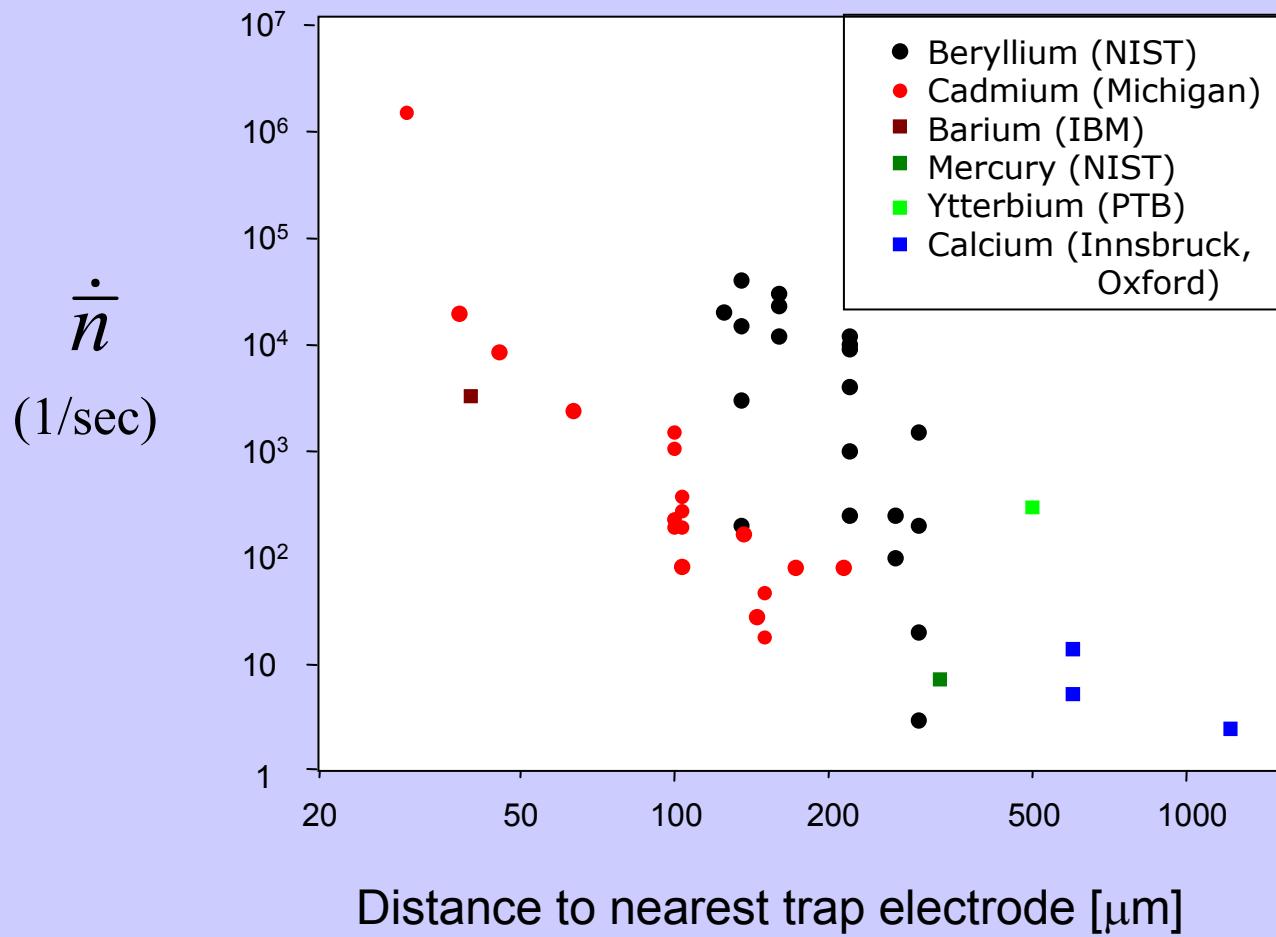
BIG PROBLEM: during the gate (at some point),  
the state of an ion qubit and motional bus state is:

$$\Psi = \alpha |\downarrow\rangle|0\rangle_m + \beta |\uparrow\rangle|1\rangle_m$$

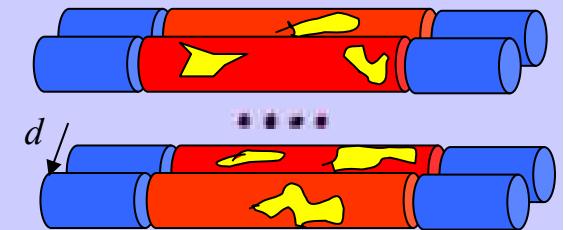


Decoherence Kills the Cat

# Heating history in 0.6-6.0 MHz traps

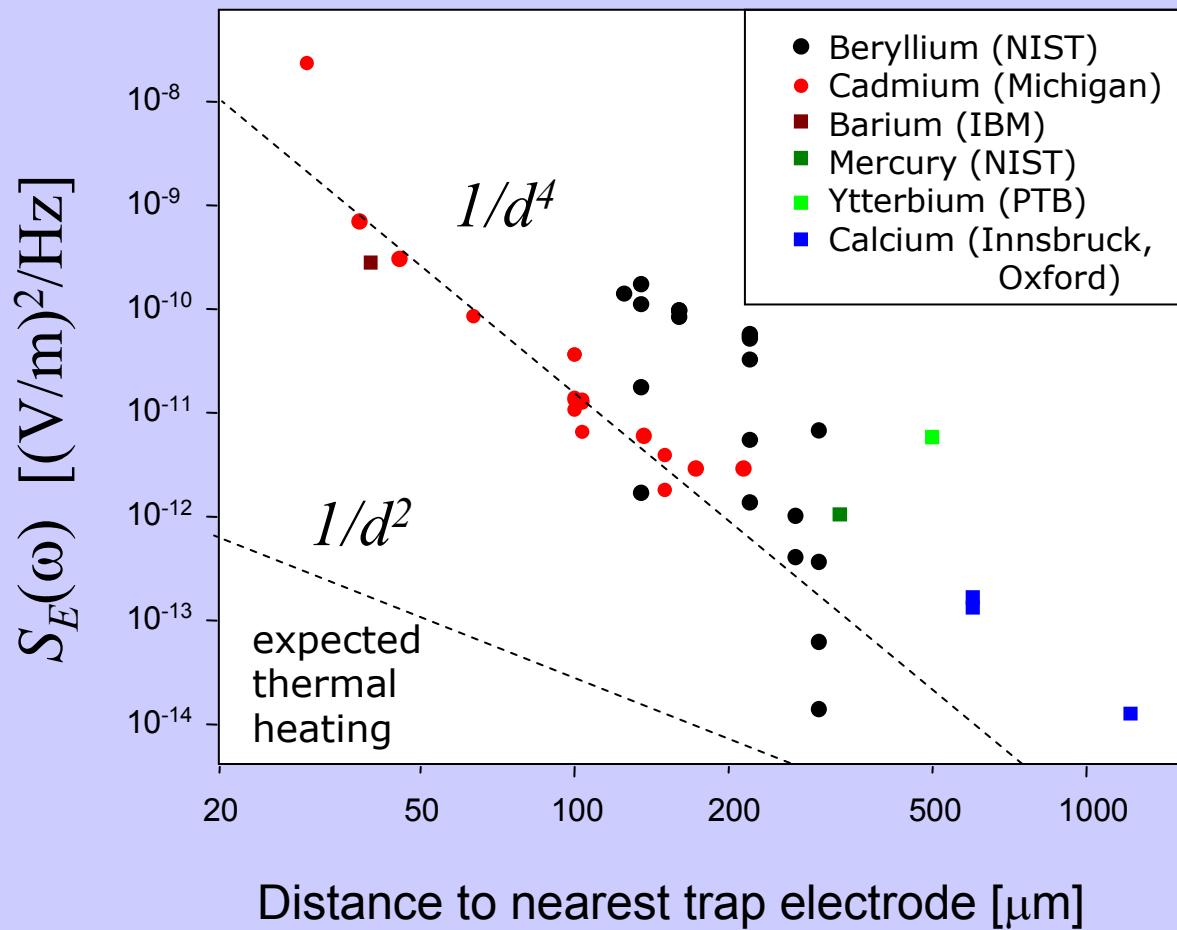


Heating due to  
fluctuating patch  
potentials (?)

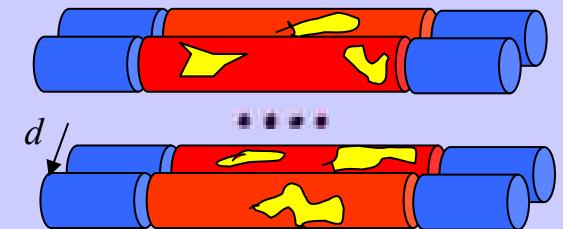


$$\dot{n} = \frac{e^2}{4m\hbar\omega} S_E(\omega)$$
$$\sim 1/d^4$$

# Electric Field Noise History in 0.6-6.0 MHz traps



Heating due to  
fluctuating patch  
potentials (?)

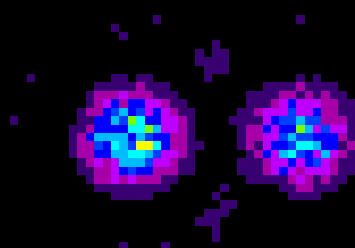


$$\dot{n} = \frac{e^2}{4m\hbar\omega} S_E(\omega) \sim 1/d^4$$

# Entangling Gate Schemes for Trapped Ion

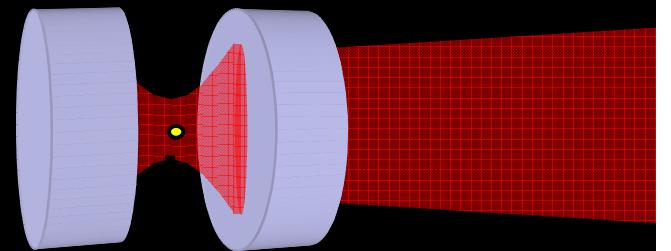
## MOTIONAL GATES

- Direct Phonon Coupling
- Spin-dependent Forces



## PHOTONIC GATES

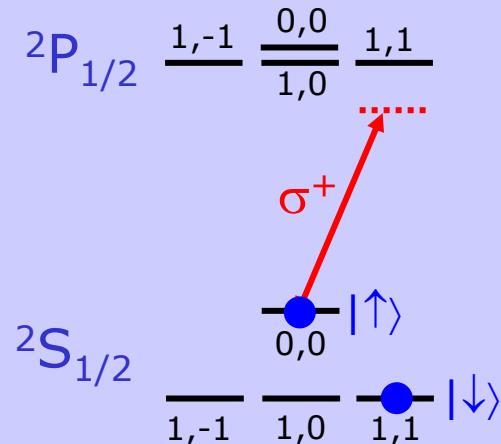
- Linear Optics (probabilistic)
- Cavity-QED



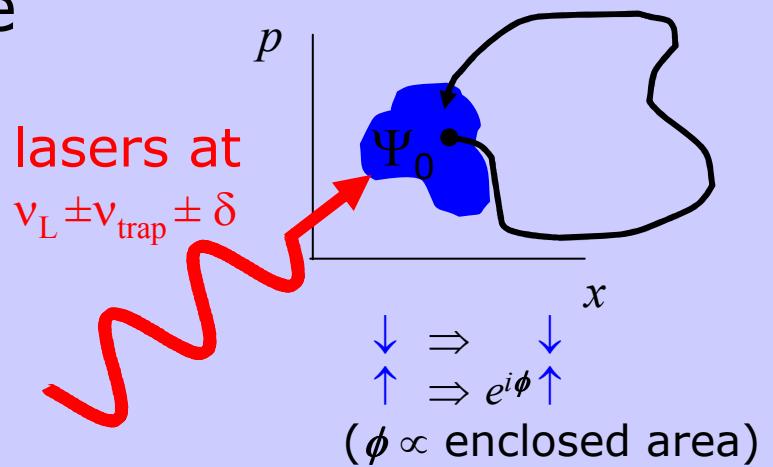
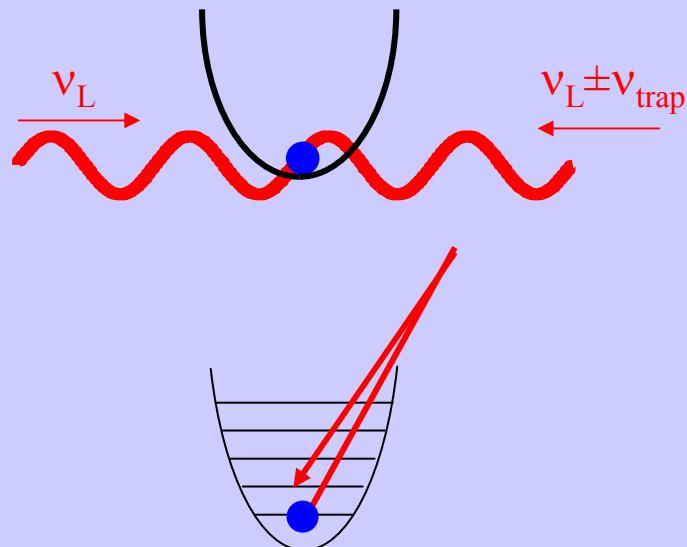
# BETTER: Indirect use of motion for quantum gates

spin-dependent force:

$$\text{Force} = F_0 |\uparrow\rangle\langle\uparrow|$$



(a) near-resonant dipole force



*Independent of initial state  
but requires Lamb-Dicke limit*

N=2 ions

e.g., force on stretch mode only



$$\begin{array}{l} \downarrow\downarrow \Rightarrow \downarrow\downarrow \\ \downarrow\uparrow \Rightarrow e^{i\phi}\downarrow\uparrow \\ \uparrow\downarrow \Rightarrow e^{i\phi}\uparrow\downarrow \\ \uparrow\uparrow \Rightarrow \uparrow\uparrow \end{array}$$

$$H = \hbar\Omega\hat{\sigma}_z^2$$

$\phi = \pi/2$ :  $\pi$ -phase gate

Mølmer and Sørensen, PRL **82**, 1835 (1999)

Solano, de Matos Filho, Zagury, PRA **59**, R2539 (1999)

Milburn, Schneider, James, Fortschr. Phys. (2000)

Leibfried *et al.*, Nature **422**, 412 (2003)

Lucas, Steane, *et al.* (2005)

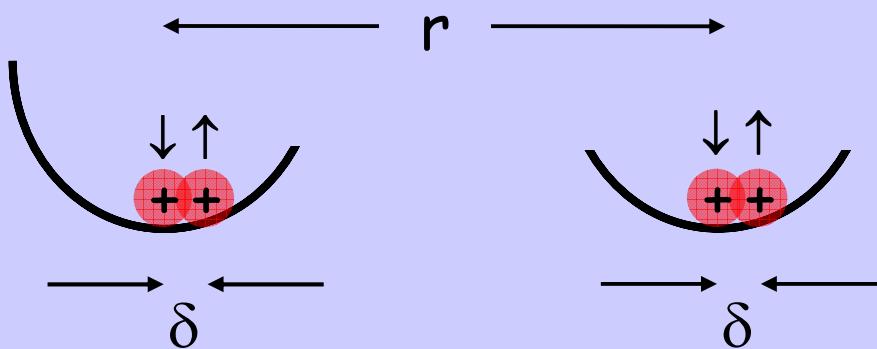
N=6 ions

$$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow + \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$$

Leibfried *et al.*, Nature **438**, 639-642 (2005)

**POSTER M25**

## (b) spin-dependent “push” force



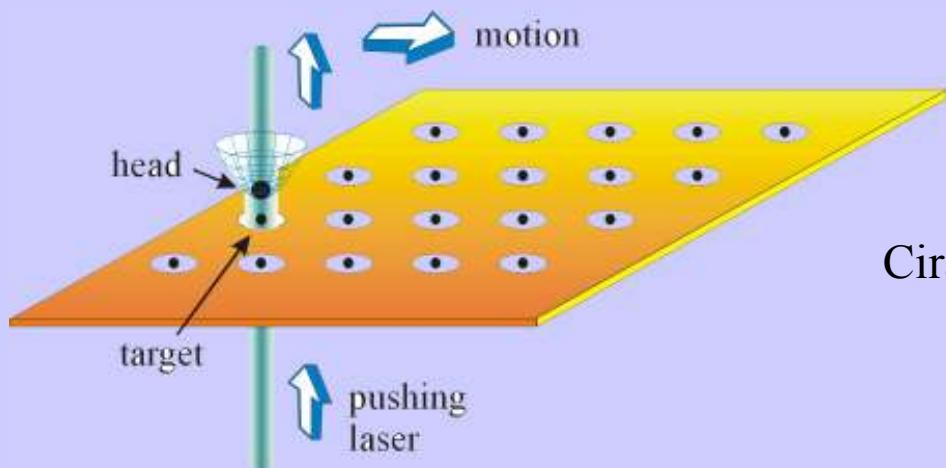
$$U_{dd} = \mu_1 \mu_2 / r^3 = (e\delta)^2 / r^3$$

quantum phase gate

$$\begin{aligned} |\downarrow\downarrow\rangle &\rightarrow |\downarrow\downarrow\rangle = |\downarrow'\downarrow'\rangle \\ |\downarrow\uparrow\rangle &\rightarrow e^{+i\kappa-i\phi/2} |\downarrow\uparrow\rangle = |\downarrow'\uparrow'\rangle \\ |\uparrow\downarrow\rangle &\rightarrow e^{-i\kappa-i\phi/2} |\uparrow\downarrow\rangle = |\uparrow'\downarrow'\rangle \\ |\uparrow\uparrow\rangle &\rightarrow |\uparrow\uparrow\rangle = e^{i\phi} |\uparrow'\uparrow'\rangle \end{aligned}$$

$\kappa$  = linear shift

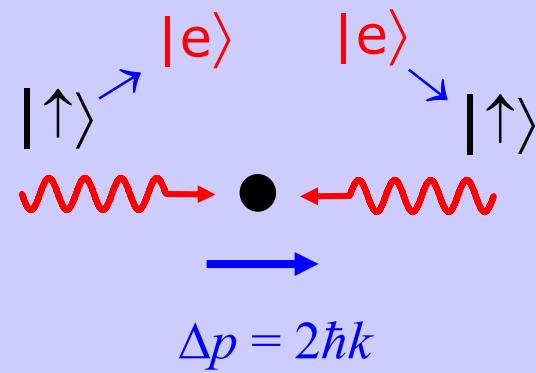
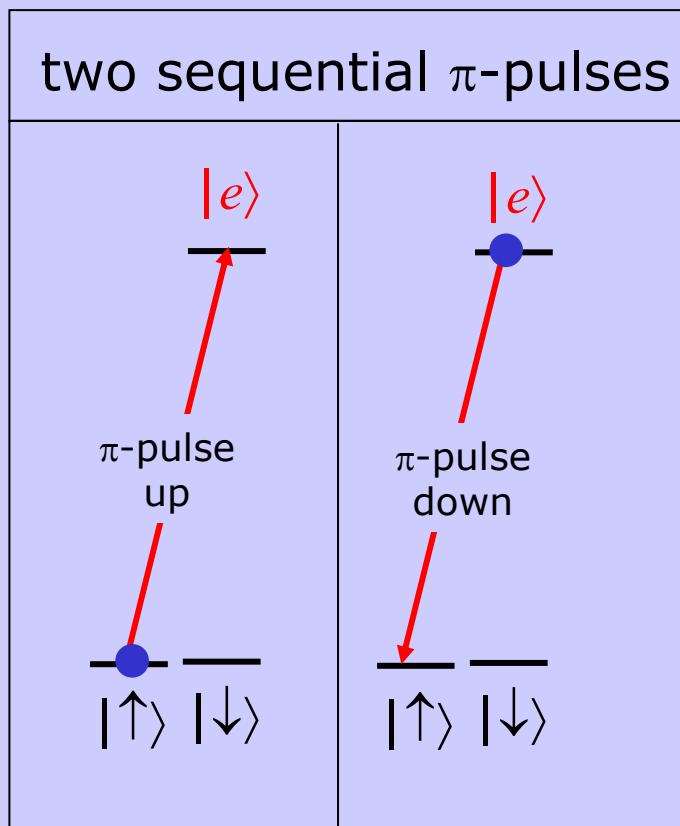
$\phi$  = nonlinear shift =  $2U_{dd}\tau/\hbar$



Cirac & Zoller *Nature* **404**, 579-581 (2000)

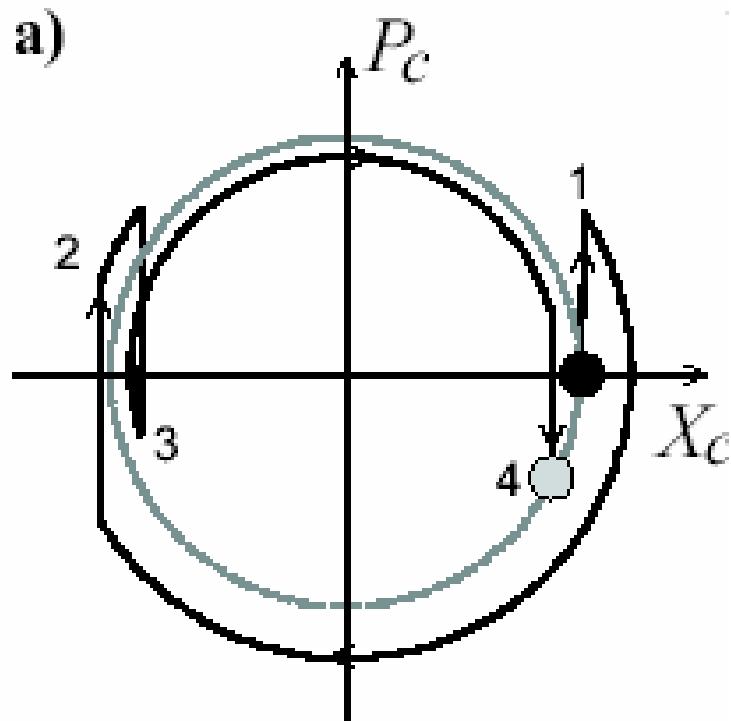
### (c) Impulsive spin-dependent forces

Poyatos, Cirac, Blatt & Zoller, *PRA* **54**, 1532 (1996)  
Garcia-Ripoll, Zoller, & Cirac, *PRL* **91**, 157901 (2003)



$$U = |\uparrow\rangle\langle\uparrow| e^{2i\eta(a+a^\dagger)}$$

*spin-dependent impulse*

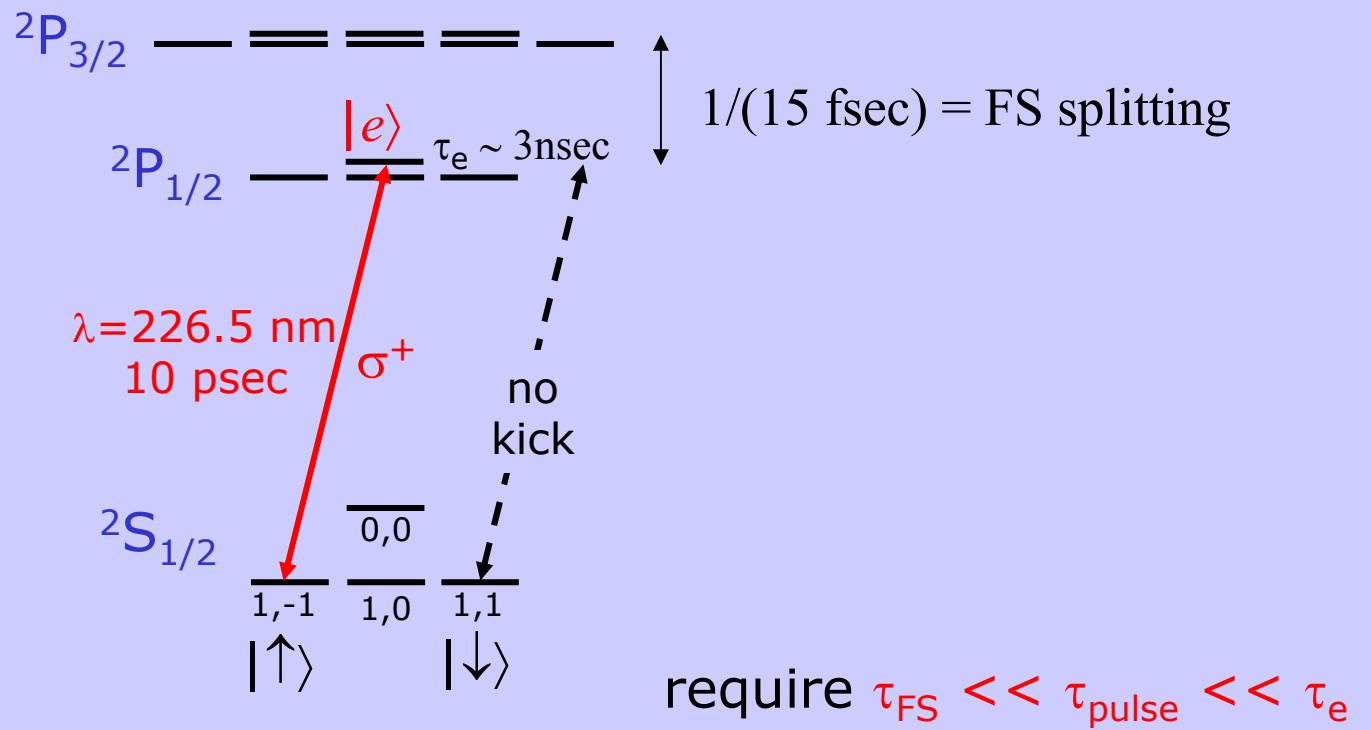


The trajectory of a normal motional mode of two ions in phase space under the influence of four photon kicks. Gray curve: free evolution. Black curve: four impulses kick the trajectory in phase space, with an ultimate return to the free trajectory after  $\sim 1.08$  revolutions.

Fast version of  $\sigma_z$  phase gate

**does not require Lamb-Dicke regime!**

e.g.  $^{111}\text{Cd}^+$



*requires ultrafast laser control*

## Problem with $\sigma_z$ gates:

With “clock” state qubits  
(no differential Zeeman shift),...

cannot realize a spin-dependent force  
(no differential AC Stark shift)

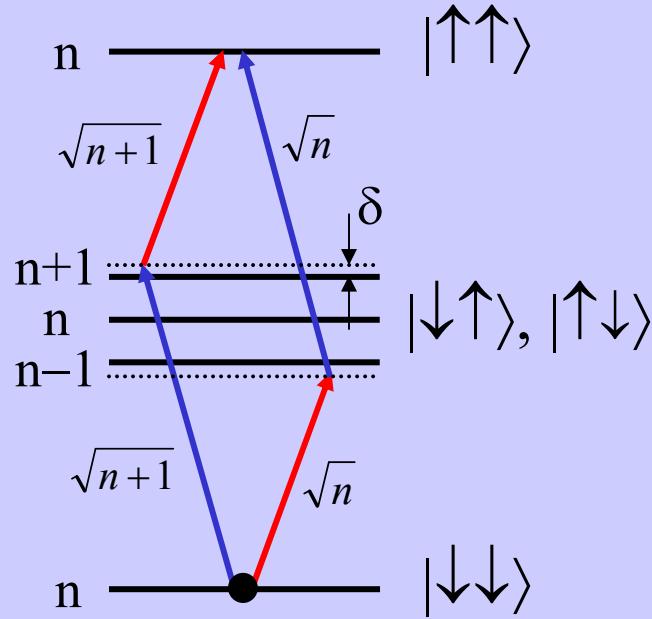
P. J. Lee, et al., Journal of Optics B **7**, S371 (2005).

## Solution:

Apply spin-dependent forces in a different basis

Mølmer and Sørensen, PRL **82**, 1835 (1999)  
Solano, de Matos Filho, Zagury, PRA **59**, R2539 (1999)  
Milburn, Schneider, James, Fortschr. Phys. (2000)

# Bichromatic coupling to *sidebands* “Molmer-Sorensen gate”



$$\begin{aligned}\Omega = \text{Rabi Freq} &= \frac{(kx_0g\sqrt{n+1})^2}{\delta} + \frac{(kx_0g\sqrt{n})^2}{-\delta} \\ &= \frac{(kx_0g)^2}{\delta} \quad \text{independent of motion !}\end{aligned}$$

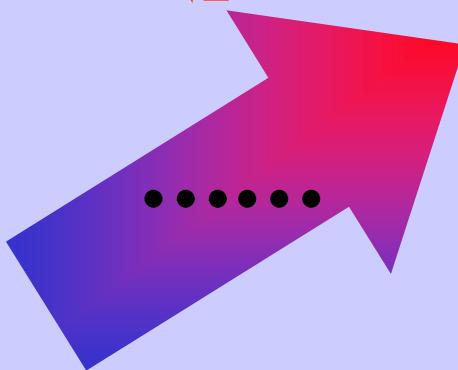
$$H = \hbar\Omega\hat{\sigma}_x^2$$

(as long as  $kx_0\sqrt{n+1} \ll 1$ : “Lamb-Dicke regime”)

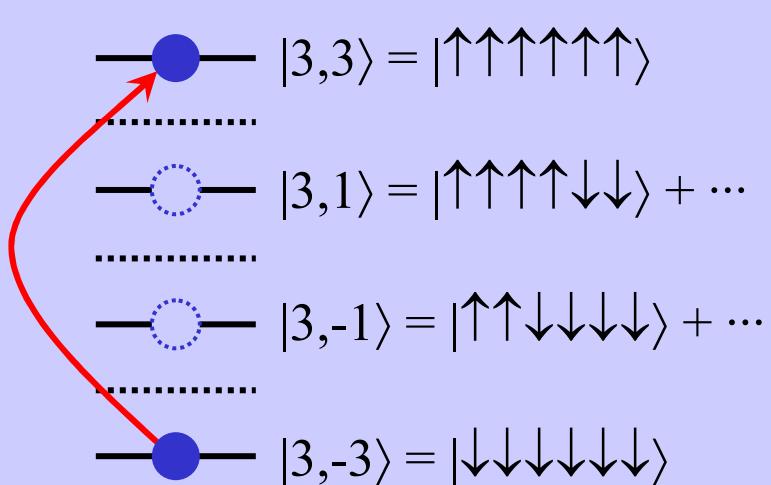
Can scalable to arbitrary N

$$|\downarrow\downarrow\downarrow\dots\downarrow\rangle \Rightarrow \frac{|\downarrow\downarrow\downarrow\dots\downarrow\rangle + |\uparrow\uparrow\uparrow\dots\uparrow\rangle}{\sqrt{2}}$$

e.g., 6 ions



$|J, J_z\rangle$



N=4 ions

Sackett, et al., *Nature* **404**, 256 (2000)

N=2 ions (clock qubits)

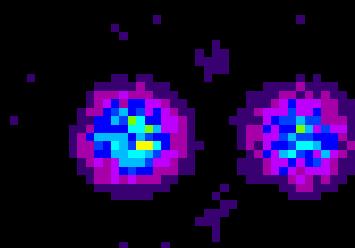
Haljan, et al, PRA **72**, 062316 (2005)

**POSTER M21**

# Entangling Gate Schemes for Trapped Ion

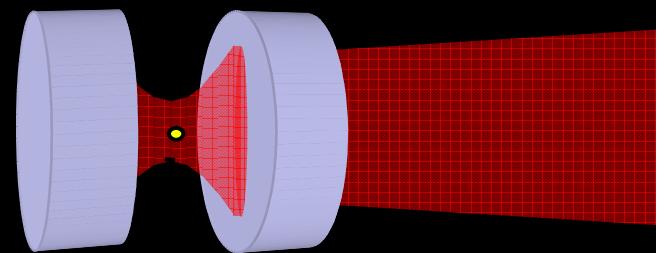
## MOTIONAL GATES

- Direct Phonon Coupling
- Spin-dependent Force

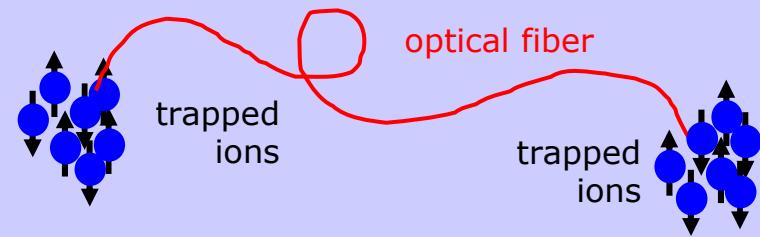


## PHOTONIC GATES

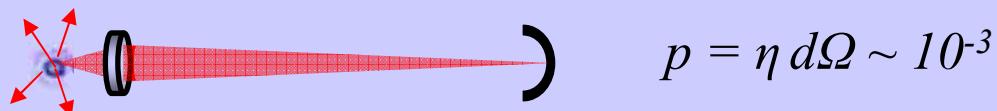
- Linear Optics (probabilistic)
- Cavity-QED



# Interfacing Trapped Ions and Photons



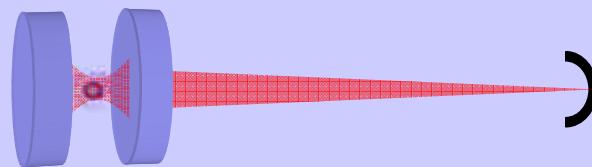
- Free space (probabilistic  $p \ll 1$ )



$$p = \eta d\Omega \sim 10^{-3}$$

- “Bad” cavities (probabilistic  $p \sim 0.3$ )

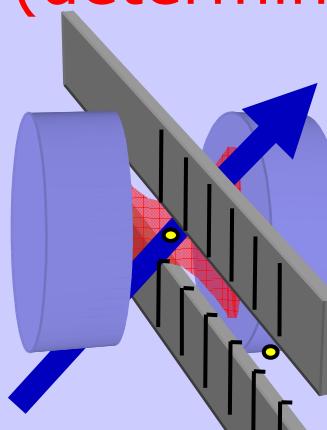
$$C = \frac{g^2}{\kappa\gamma} \approx 1$$



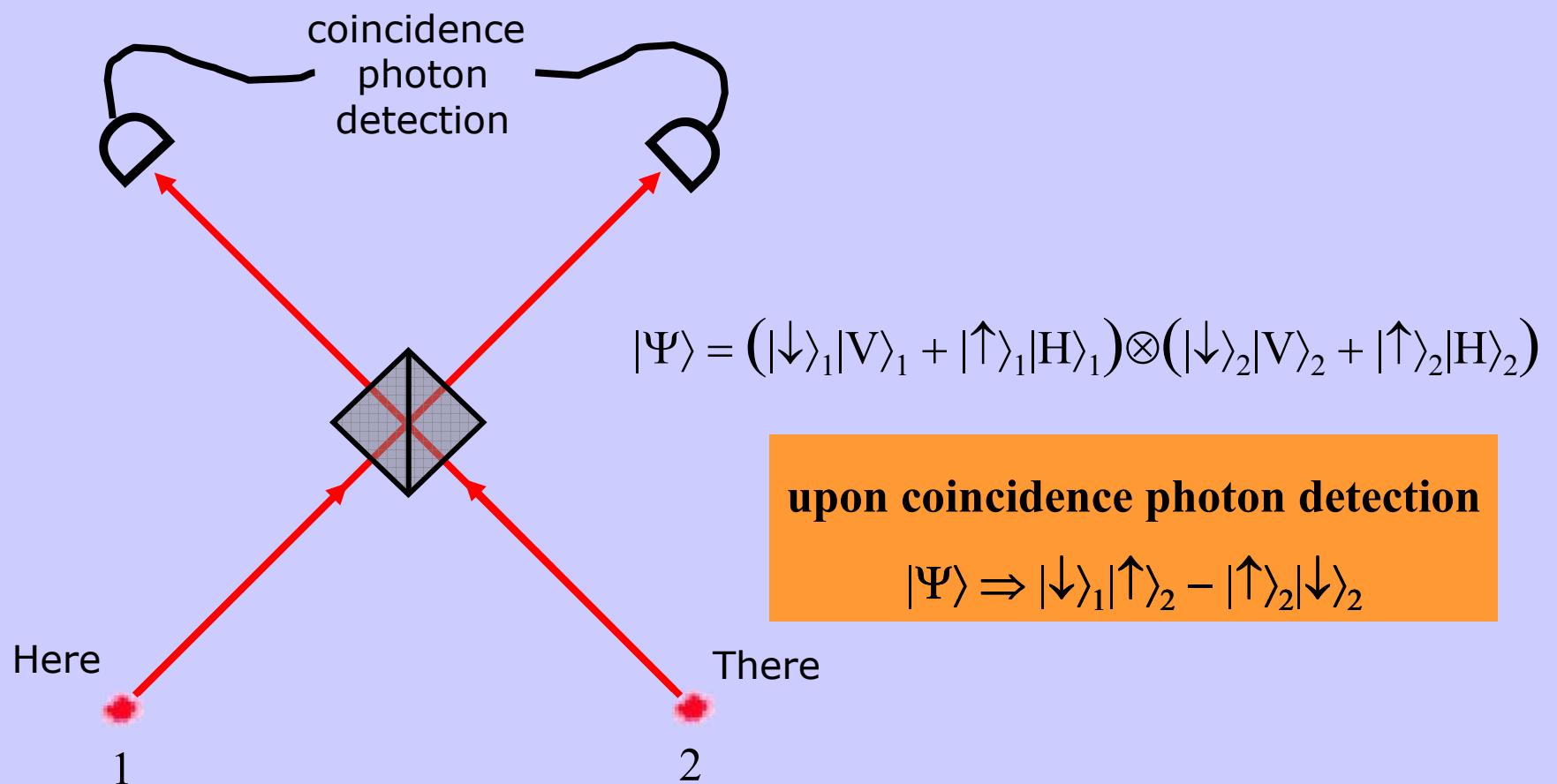
$$p = \eta \left( \frac{C}{C+1} \right) \approx 0.1 - 0.3$$

- Strong coupling (deterministic cavity-QED)

$$C = \frac{g^2}{\kappa\gamma} \gg 1$$



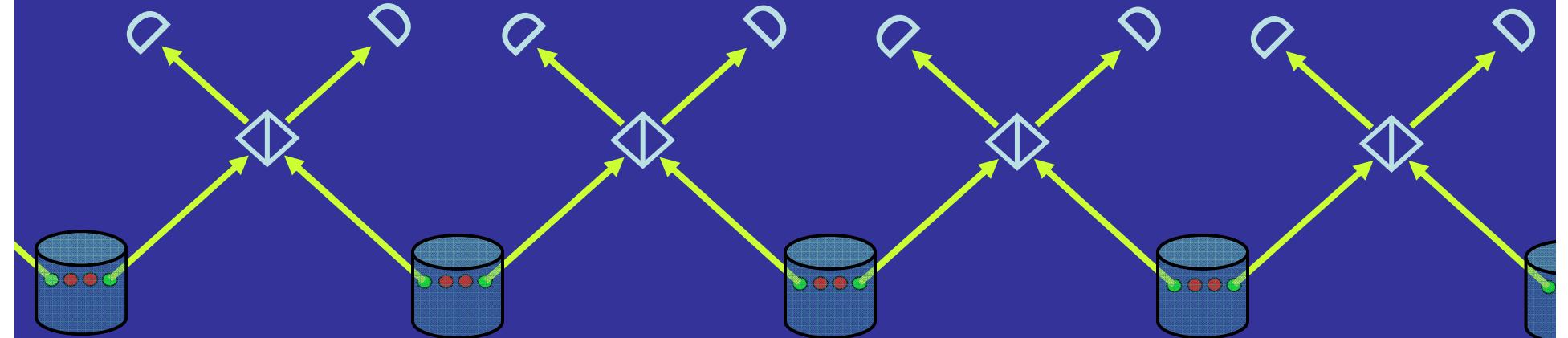
# Networking distant ions



insensitive to interferometric phase noise  
insensitive to ion motion

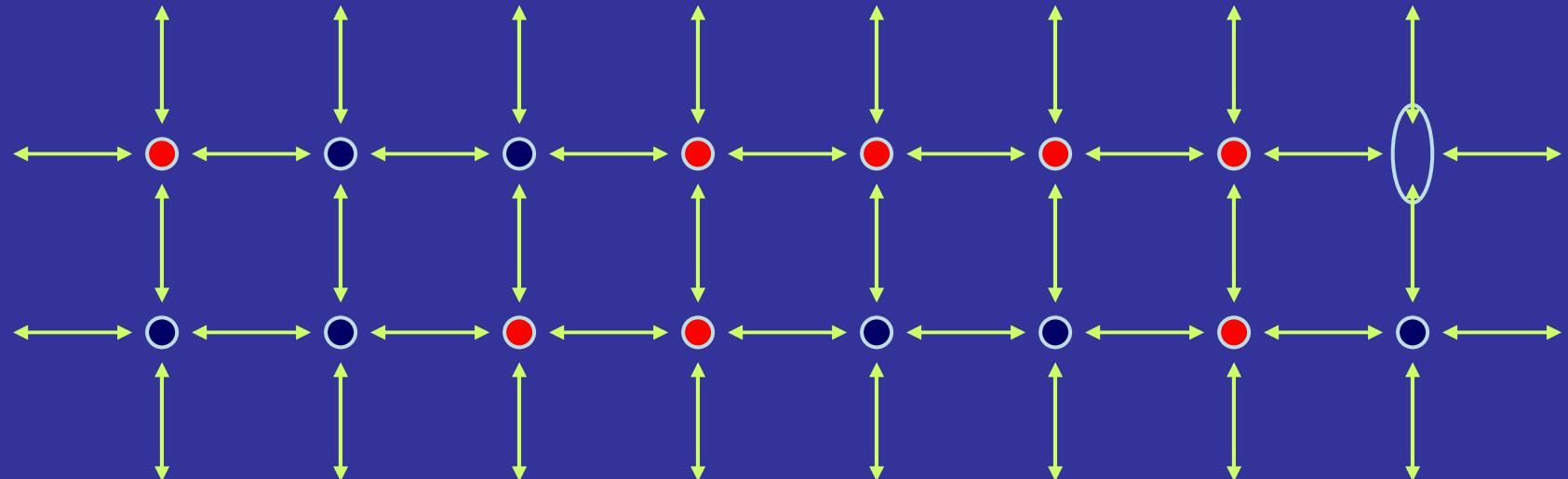
Simon & Irvine, PRL 91, 110405 (2003)  
Blinov, et al., Nature 428, 153 (2004)  
Duan, et. al., QIC 4, 165 (2004)  
**POSTERS M04, M17, M18**

# Quantum networking with probabilistic entanglement



Quantum repeater network

Briegel *et al.*, PRL **81**, 5932 (1998)



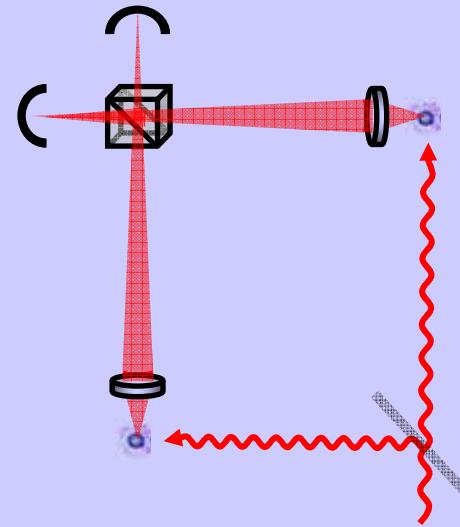
Cluster state quantum computing

Raussendorf and Briegel, PRL **86**, 910 (2001)

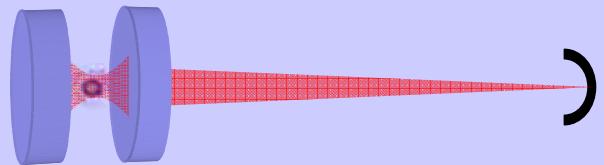
Duan and Raussendorf, PRL **95**, 080503 (2005)

- Free space (probabilistic  $p \ll 1$ )

$$\text{Pair entanglement rate} = Rp^2 \sim 1 \text{ Hz}$$



- Optical Cavities around ions

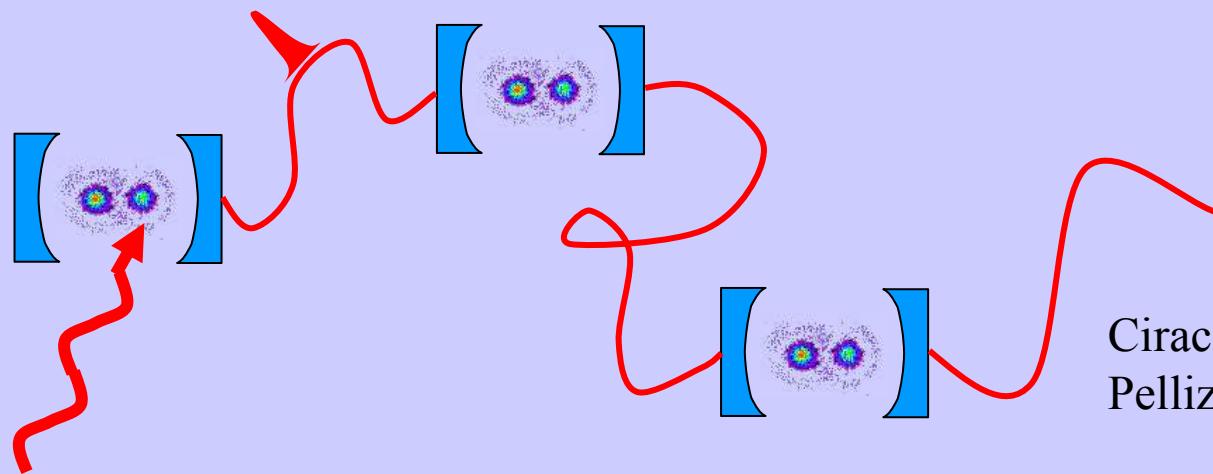


$$C = \frac{g^2}{\kappa\gamma} \gg 1 \quad g = \frac{\mu E}{\hbar} \sim \frac{1}{\sqrt{Vol}}$$

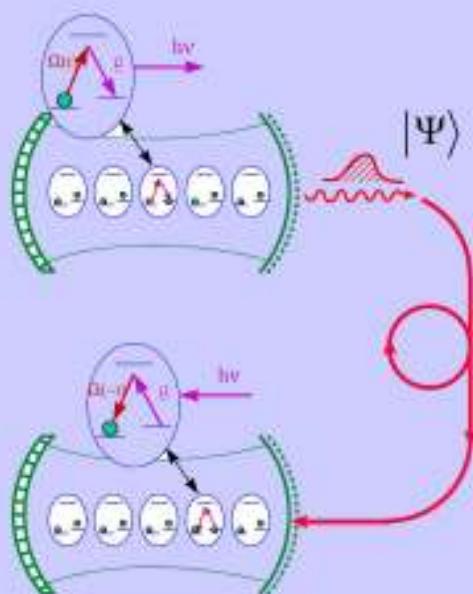
G. Guthorlein, M. Keller, K. Hayasaka, W. Lange, and H. Walther,  
 “A single ion as a nanoscopic probe of an optical field”  
*Nature* **414**, 49 (2001).

Mundt, A. Kreuter, C. Becher, D. Leibfried, J. Eschner, F. Schmidt-Kaler, R. Blatt,  
 “Coupling a Single Atomic Quantum Bit to a High Finesse Optical Cavity”  
*Phys. Rev. Lett.* **89**, 103001 (2002).

# Deterministic coupling of atoms to a single-mode cavity



Cirac, Zoller, Pellizzetti, PRL (1995)  
Pellizzetti, PRL **79**, 5242 (1997)

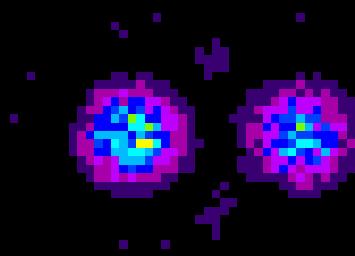


Cirac, Zoller, Kimble, Mabuchi, PRL **78**, 3221 (1997).

# Entangling Gate Schemes for Trapped Ion

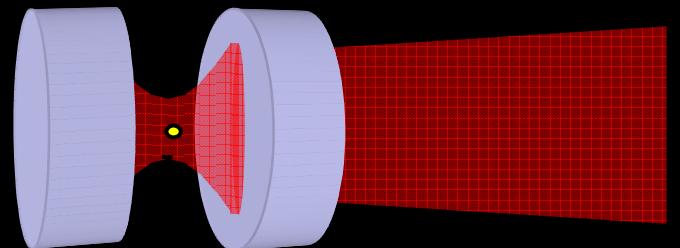
## MOTIONAL GATES

- Direct Phonon Coupling
- Spin-dependent Force



## PHOTONIC GATES

- Linear Optics (probabilistic)
- Cavity-QED



## COUPLING TO SOLID-STATE?

- superconducting
- quantum dots